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Phase structure of nuclear matter in a model with hybrid derivative coupling of scalar meson or equivalently nonlinear self interaction of meson

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Abstract : We have shown equivalence of recently proposed hybrid derivative coupling of scalar mesons σ to highly nonlinear self interactions of σ mesons in the study of thermal properties of nuclear matter Characteristics of both liquid gas phase transition and phase transition of hadronic matter to quark gluon plasma have been investigated

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In this work, we intend to show that the general form of hybrid derivative coupling of scalar meson σ to nucleon in our recently proposed model [1] of nuclear matter, is equivalent to one having usual Yukawa point coupling and nonlinear self interactions of scalar mesons involving $\sigma^3, \sigma^4, \cdots$ terms. The coefficients of $\sigma^3, \sigma^4, \cdots$ terms in the above case are all positive unlike the coefficient of σ^4 terms in Boguta's [2] model which is sometimes negative causing some conceptual difficulty [3]. It is of interest to investigate some properties of nuclear matter at finite temperature like liquid gas phase transition, variation of effective nucleon mass M^* at high density ρ and high temperature T and also phase transition from hot dense nuclear matter to quark gluon plasma (QGP) [4] in the frame work of above mentioned model which has been previously used to study nuclear matter at zero temperature [1]. In the above mentioned hybrid model [1] characterized by a hybridization parameter α, the strength of Yukawa point coupling ($\alpha = 0$) and that of derivative coupling [5] ($\alpha = 1$) is taken in the ratio $(1-\alpha)/\alpha$. Suitable value of α is chosen which yields satisfactory results for bulk properties of nuclear matter.

The Lagrangian density L of above mentioned hybrid model can be put into the following form of transformed L [1] involving fields of baryons B (ψ_B), scalar (σ) and vector (ω_μ) mesons using notations of reference [1,5].

$$L = \sum_B \bar{\psi}_B\left(i\gamma^\mu\partial_\mu - M_B^* - g_{vB}\gamma^\mu\omega_\mu\right)\psi_B + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\right)$$

$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_v^2\omega^\mu\omega_\mu, \quad (1)$$

obtained by resealing the baryon wave function

$$\psi_B \to \left(1 + \alpha\sigma g_{sB}/M_B\right)^{-1/2}\psi_B. \quad (2)$$

The effective baryon mass M_B^* in (1) is given by

$$M_B^* = M_B\left[1 - (1-\alpha)\sigma g_{sB}/M_B\right]/\left[1 + \alpha\sigma g_{sB}/M_B\right]. \quad (3)$$

We assume [1]

$$g_{sB}/M_B = g_{sN}/M_N = g_s/M \text{ (say)}, \quad (4)$$

where 'B' stands for any baryon like nucleon (N) and delta resonance (Δ). We also take [1]

$$g_{vB} = g_{vN} = g_v \text{ (say)}, \quad (5)$$

It has been shown in Ref. [1] that the part of the Lagrangian given by (1) involving interaction between scalar meson and baryon which is of the form

$$L_{\text{int},\sigma} = \sum_B g_{sB}\sigma\left(1 + \alpha g_s\sigma/M_B\right)\bar{\psi}_B\psi_B, \quad (6)$$

appears to have highly non linear coupling between scalar mesons and baryons.

Let us define transformed scalar field ($\tilde{\sigma}$) by

$$\tilde{\sigma} = \sigma/\left[1 + \alpha g_s\sigma/M\right]. \quad (7)$$

So that we have the following simple relation for effective baryon mass

$$M_B^* = M_B\left[1 - \left(g_{sB}/M_B\right)\tilde{\sigma}\right]. \quad (8)$$

Then the part of the transformed Lagrangian concerning interaction involving transformed scalar meson field $\tilde{\sigma}$ and baryon is of the form

$$L_{\text{int},\tilde{\sigma}} = \sum_B g_{sB}\tilde{\sigma}\bar{\psi}_B\psi_B$$

as in Boguta's model [2].

In mean field theory (MFT), approximation the part of the Lagrangian for uniform nuclear matter involving new scalar field σ, takes the following form

$$
\begin{aligned}
-L_{\tilde{\sigma}} &= \frac{1}{2} m_s^2 \left[\tilde{\sigma} / \left(1 - \alpha g_s \tilde{\sigma}/M\right)\right]^2 \\
&= \frac{1}{2} m_s^2 \tilde{\sigma}^2 + m_s^2 \alpha g_s \tilde{\sigma}^3/M + \frac{3}{2} m_s^2 \alpha^2 g_s^2 \tilde{\sigma}^4/M^2 + \cdot
\end{aligned} \tag{10}
$$

which contains a series of self interaction terms.

In Boguta's model, the corresponding $-L_{\tilde{\sigma}}$ contains self interaction terms involving $\tilde{\sigma}^3$ and $\tilde{\sigma}^4$ only. For effective nucleon mass M^* in the range 0.6 M to 0.7 M [6] and recent estimate for incompressibility $K = 300 \pm 25$ MeV [7], it is found that the coefficient of $\tilde{\sigma}^4$ term is positive but that of σ^4 term occurring in $-L_\sigma$ of Boguta's model is negative [8] implying no lower bound in energy spectrum [3]. No such conceptual difficulty exists in the hybrid model or its transformed version.

In MFT the field equations for scalar meson $\tilde{\sigma}$ and vector meson are

$$
m_s^2 \tilde{\sigma} / \left(1 - \alpha g_s \tilde{\sigma}/M\right)^3 = g_s \sum \left(M_B/M\right)\left\langle \overline{\psi}_B \psi_B \right\rangle \tag{11}
$$

and

$$
\omega_0 = \left(g_v/m_v^2\right) \sum_B \left\langle \psi_B^+ \psi_B \right\rangle. \tag{12}
$$

Expressions for energy density ε and P are given by

$$
\begin{aligned}
\varepsilon &= \frac{1}{2} m_s^2 \tilde{\sigma}^2 / \left(1 - \alpha g_s \tilde{\sigma}/M\right)^2 + \frac{1}{2} m_v^2 \omega_0^2 \\
&+ \sum_B \frac{\gamma_B}{(2\pi)^3} \int d^3k E_{Bk}^* \left[n_{Bk}(T) + \bar{n}_{Bk}(T)\right]
\end{aligned} \tag{13}
$$

and

$$
\begin{aligned}
P &= -\frac{1}{2} m_s^2 \tilde{\sigma}^2 / \left(1 - \alpha g_s \tilde{\sigma}/M\right)^2 + \frac{1}{2} m_v^2 \omega_0^2 \\
&+ \frac{1}{3} \sum \frac{\gamma_B}{(2\pi)^3} \int d^3k \frac{k^2}{E_{Bi}^*} \left[\eta_{Bk}(T) + \bar{n}_{Bk}(T)\right]
\end{aligned} \tag{14}
$$

where

$$
n_{Bk}(T), \bar{n}_{Bk}(T) = \left[\exp\left(E_{Bk}^* \mp \mu'\right)/T + 1\right]^{-} \tag{15}
$$

and

$$
E_{Bk}^* = \left(M_B^{*2} + k^2\right)^{1/2}. \tag{16}
$$

The chemical potential μ can be expressed as

$$
\mu = \mu' + g_v \omega_0. \tag{17}
$$

The characteristics of liquid gas phase transition occurring at low density and low temperature, like critical temperature T_{cr} and critical density ρ_{cr}, are determined from the condition

$$dp/d\rho = d^2p/d\rho^2 = 0 \text{ at } T = T_{cr} \text{ and } \rho = \rho_{cr} \text{ [9].}$$

There is a connection between liquid gas phase transition and formation of fragments in nuclear collision. The above characteristics for liquid gas phase transition are found to be $T_{cr} = 18.2$ MeV $\rho_{cr} = 0.4\,\rho_0$ for $\alpha = 0$ (Walecka model [10]), $T_{cr} = 17.5$ MeV, $\rho_{cr} = 0.36\,\rho_0$ for $\alpha = 1/4$ and $T_{cr} = 16.5$ MeV $\rho_{cr} = 0.33\,\rho_0$ for $\alpha = 1$ (purely derivative coupling model [5]) where ρ_0 is the saturation density. It has been discussed in Ref. [1] that the model for hybridization parameter $\alpha = 1/4$ may give reasonable results for bulk properties of nuclear matter.

The effective nucleon mass M_N^* (or M^*) of delta excited nuclear matter can be evaluated from (8) and (11). For large density and $T = 0$, this can be expressed as [1].

$$M^*/M \cong \left(A/(1-\alpha)^3\right)\left(M/k_F\right) \quad \text{for } \alpha \neq 1 \tag{18}$$

$$\cong A^{1/4}\left(M/k_F\right)^{1/2} \quad \text{for } \alpha = 1 \tag{19}$$

Similar relations for M^* at high T and zero density are given below

$$M^*/M \cong \left(3/\pi^2\right)\left(A/(1-\alpha)^3\right)\left(M/T\right)^2 \quad \text{for } \alpha \neq 1 \tag{20}$$

$$\cong \left(3A/\pi^2\right)^{1/4}\left(M/T\right)^{1/2} \quad \text{for } \alpha = 1 \tag{21}$$

where
$$A = \left(4\pi^2/C_s^2\right)\left(\gamma_N + (M_\Delta/M_N)^2\gamma_\Delta\right)^{-1} \tag{22}$$

In the relation (21), γ_N and γ_Δ are degeneracy factors for nucleons and delta resonances respectively. It appears that effective nucleon mass M^* falls off more slowly as α is enhanced. Now it can be shown that incompressibility K is partly dependent on $\partial M^*/\partial\rho$ at $\rho = \rho_0$. So we can expect that K decreases with increasing α and this is observed in Ref. [1].

At high temperature or at high density, nuclear matter undergoes a phase transition to quark gluon plasma and this is expected to occur in heavy ion collision. Using in this case, Gibbs criteria [4] of thermal, chemical and pressure equilibrium between the two phases, it is found that phase transition temperature $T_{ph} = 125.6$ MeV (considering nucleons and pion gas) for $B^{1/4} = 178$ MeV [11] and $T_{ph} = 190$ MeV for $B^{1/4} = 236$ MeV [12] in the case of delta excited nuclear matter. On the other hand, phase transition density ρ_{ph} for delta excited hadronic matter of zero temperature changes from 8 ρ_0 to 8.75 ρ_0 as $B^{1/4}$ is increased from 178 MeV to 236 MeV where B is the bag constant in MIT bag model [13]. In the above case, $\alpha = 1/4$. Further, above mentioned phase transition density for hadronic matter when $B^{1/4} = 236$ MeV, is quite close to the corresponding findings of some recent investigations [3].

It is interesting to find that hybrid derivative coupling model or highly nonlinear coupling of scalar meson to baryons, can be interpreted in terms of nonlinear self interaction terms of scalar meson which also appear in Boguta's model. But Boguta's model has the undesirable feature of negative sign of σ^4 term which is physically untenable since the potential [14] takes infinitely large negative value as σ is greatly enhanced. It may be mentioned that the above mentioned hybrid derivative coupling model can also be identified with Walecka model having density-dependent coupling constant [1]. The most appropriate value of hybridization parameter α and bag constant B should be fixed from more reliable results for properties of nuclear matter at saturation, characteristics of liquid gas phase transition, and hadronic phase transition density in the first order phase transition from nuclear matter to QGP.

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